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Abelian decomposition method for G_2 gauge theory

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Abstract. The method of Abelian decomposition proposed by Faddeev and Niemi is used to derive the low-energy effective lagrangian of G_2 gauge theory. The G_2 algebra is studied. The commutation relations among the generators of the G_2 algebra are established, based on the framework of its regular maximal subalgebra, an $SU(3)$ algebra.

It is well-known that a Yang–Mills gauge theory at high energy describes weakly interacting massless gluons and can be solved perturbatively thanks to asymptotic freedom. At low energy, the theory becomes strongly coupled and the method of perturbation fails. Some non-perturbative techniques have to be developed to tackle this problem. A quantitative explanation of the problem is as follows. At low energy, the Yang–Mills gauge theory exhibits color confinement due to the dual Meissner effect. The dynamics of this effect will take place when the gauge theory is Abelian projected to its maximal Abelian subgroup [1,2]. Consequently, the spectrum of the low-energy theory would possess massive composites of gauge fields such as glueballs.

A systematic method of Abelian decomposition that is used to parameterize the four-dimensional $SU(2)$ Yang–Mills connection was proposed by Faddeev and Niemi [3][1]. In the decomposition, the parameterized connection contains a set of new variables, which is appropriate for describing the theory in the infrared limit. It is shown that, at a certain low-energy phase, the decomposed theory becomes the so-called Faddeev–Skyrme model. This model is known to support topological solutions that can be regarded as candidates for glueballs [3,5]. The topological aspects of the Abelian decomposed model are investigated in detail in [6]. Moreover, the Hamiltonian structure of the Faddeev–Skyrme model has been studied and is found to coincide with a symmetrical top rotating in the $SU(2)$ space [7]. The generalization of the Abelian decomposition method to the $SU(N)$ Yang–Mills theory is reported in [8,9]. Similarly, the method is also extended to the general cases of $SO(N)$ and $Sp(2N)$ Yang–Mills theories [10].

In the present letter, the G_2 algebra is examined systematically. The generators are classified based on its regular maximal subalgebra, an $SU(3)$ algebra. The commutation relations and structure constants of the G_2 algebra are established using the same $SU(3)$ symmetry. After discussion of the G_2 group, we focus our attention on the Abelian decomposition for the four-dimensional G_2 Yang–Mills theory. Twelve gauge covariant one-forms that determine a basis of roots for the G_2 algebra are constructed. Using these covariant one-forms and other dual variables, the G_2 gauge connection is completely Abelian decomposed via the Faddeev–Niemi method. Hence, the low-energy effective lagrangian of the G_2 Yang–Mills theory can be derived straightforwardly.

The rank of the exceptional Lie group G_2 is 2 and its Lie algebra contains 14 generators. Let us denote them by $\mathcal{T}_\alpha$ for $\alpha = 1$ to 14. On the root-vector diagram, these 14 generators are divided into three categories: two null roots, six longer roots and six shorter roots. All together, they form a highly symmetrical diagram, the customary "Star of David." The regular maximal subalgebra of G_2 is an $SU(3)$ algebra. It is not difficult to see that under the action of $SU(3)$, the 14 generators transform like $8 \oplus 3 \oplus \bar{3}$. Based on this fact, we further denote the 14 generators of the G_2 group as follows: the commuting Cartan subalgebra by T_i for $i = 3$ and 8, the longer roots by T_a and the shorter roots by t_a, where the subscripts a in both cases take values in the set $(1, 2, 4, 5, 6, 7)$.

Obviously, under such an arrangement the combined generators $T_A = (T_i, T_a)$ generate the $SU(3)$ algebra. That is,

$$[\,T_A, T_B\,] = \mathrm{i}\, f_{ABC}\, T_C\,. \tag{1}$$

Here, the f_{ABC} $(A, B, C = 1$ to $8)$ are the standard $SU(3)$ structure constants in the Gell-Mann basis and are antisymmetric with respect to interchange of any two indices. Furthermore, from the multiplication law of the $SU(3)$ generators,

$$T_A\, T_B = \frac{1}{2}\left[\frac{1}{3}\,\delta_{AB} + (\mathrm{i}\, f_{ABC} + d_{ABC})\, T_C\right], \tag{2}$$

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[1] Faddeev and Niemi have also proposed a modified method of Abelian decomposition for $SU(2)$ Yang–Mills theory. In this decomposition, the duality of electric and magnetic variables is explicitly realized [4]

we define two sets of matrices $(\mathcal{F}_A)_{BC} = -\mathrm{i} f_{ABC}$ and $(\mathcal{D}_A)_{BC} = d_{ABC}$. Then the following identities can be verified:

$$\mathcal{F}_i\,\mathcal{D}_j - \mathcal{D}_j\,\mathcal{F}_i = 0\,, \tag{3}$$
$$\mathcal{F}_i\,\mathcal{D}_j + \mathcal{F}_j\,\mathcal{D}_i - d_{ijk}\,\mathcal{F}_k = 0\,, \tag{4}$$

where the subscripts $(i, j, k = 3, 8)$ are those in the Cartan subalgebra.

In order to establish the other commutation relations *neatly* among the generators $\mathcal{T}_\alpha = (T_i, T_a, t_a)$, we have to deal with the shorter roots t_a appropriately. We notice that with the Cartan generator $T_3\,(T_8)$ on the root-vector diagram representing the symmetrical axis for the longer roots, the other generator $T_8\,(-T_3)$ would serve in a similar role for the shorter root system. This simple observation enables us to determine the rest of the commutation relations. They are

$$[\,T_i, t_a\,] = -\frac{\mathrm{i}}{\sqrt{3}}\,\epsilon_{ij}\,f_{jab}\,t_b\,, \tag{5}$$
$$[\,t_a, t_b\,] = \frac{2\mathrm{i}}{\sqrt{3}}\,f_{abc}\,t_c + \frac{\mathrm{i}}{\sqrt{3}}\,f_{abi}\,\epsilon_{ij}\,T_j + \mathrm{i}\,g_{abc}\,T_c\,, \tag{6}$$
$$[\,t_a, T_b\,] = -\mathrm{i}\,g_{acb}\,t_c\,, \tag{7}$$

where ϵ_{ij} is the Levi-Civita tensor with $\epsilon_{38} = -\epsilon_{83} = 1$.

In the above equations, (5), (6) and (7), we introduce one more structure constant g_{abc}, in addition to the $SU(3)$ counterparts f_{abc} and f_{abi}. The g_{abc} are antisymmetric with respect to interchange of the first two indices, that is, $g_{bac} = -g_{abc}$. The non-zero constants are listed below:

$$\begin{aligned}
&g_{147} = 1/2, \quad g_{156} = -1/2,\\
&g_{174} = -1/2, \quad g_{165} = -1/2,\\
&g_{246} = -1/2, \quad g_{257} = -1/2,\\
&g_{264} = 1/2, \quad g_{275} = -1/2,\\
&g_{462} = 1/2, \quad g_{471} = 1/2,\\
&g_{561} = 1/2, \quad g_{572} = -1/2.
\end{aligned} \tag{8}$$

It is stressed that both structure constants, f_{abc} and g_{abc}, have non-vanishing elements with the same subscript labels. There are various relations among the structure constants. For example, we have the summation identities

$$g_{cda}\,g_{cdb} = \delta_{ab} \quad \text{and} \quad f_{cda}\,g_{cdb} = 0\,. \tag{9}$$

We also list some of the relations derivable from the Jacobi identities below

$$\begin{aligned}
&f_{abd}\,g_{cde} - f_{acd}\,g_{bde} + f_{bcd}\,g_{ade} = 0,\\
&\sqrt{3}\,g_{abd}\,f_{dcj}\,\epsilon_{ji} + g_{adc}\,f_{bdi} - g_{bdc}\,f_{adi} = 0,\\
&\frac{1}{\sqrt{3}}\,\epsilon_{ij}\,f_{abj}\,f_{ice} + g_{abd}\,f_{cde} + g_{adc}\,g_{bde} - g_{bdc}\,g_{ade} = 0.
\end{aligned} \tag{10}$$

Let us briefly restate what we have shown regarding the G_2 algebra. The 14 generators ($\mathcal{T}_\alpha$ for $\alpha = 1, \cdots, 14$) of the G_2 group are classified into three parts: T_i, T_a, and t_a. These generators satisfy the commutation relations (1), (5), (6) and (7), and are normalized to

$$\mathrm{Tr}\,(\mathcal{T}_\alpha \mathcal{T}_\beta) = \frac{1}{2}\,\delta_{\alpha\beta}. \tag{11}$$

As a result, a generic G_2 Lie-algebra element v has an expansion in terms of the generators as follows: $v = v^i\,T_i + v^a\,T_a + \tilde{v}^a\,t_a$.

Now, we are readily to generalize the method of Abelian decomposition for the G_2 Yang–Mills connection one-form

$$A = A_\mu\,\mathrm{d}x^\mu = \left(A^i_\mu\,T_i + A^a_\mu\,T_a + \tilde{A}^a_\mu\,t_a\right)\mathrm{d}x^\mu. \tag{12}$$

Following the decomposition procedures presented in [9, 10], we first conjugate the elements of the Cartan subalgebra T_i by a generic element $g \in G_2$ to generate Lie-algebra valued vector fields:

$$m_i = g\,T_i\,g^{-1}, \tag{13}$$

where $i = 3, 8$. The fields m_i depend on 12 independent variables, since they remain invariant if g transforms by $g \to gh$, for h belongs to the $U(1)^2$ subgroup of G_2.

We then want to parameterize the connection one-form A using the fields m_i defined in (13). According to Cho's prescription [11], the connection one-form (12) admits this decomposition:

$$A = C^i\,m_i + \frac{1}{\mathrm{i}}\,[\,\mathrm{d}m_i, m_i\,] + (\text{covariant part}), \tag{14}$$

where C^i for $i = 1, 2$ are Abelian connection one-forms. In the decomposition (14), the first two terms by construction preserve the full G_2 gauge characteristics. Hence, the variables appearing in the (covariant part) must transform covariantly under a G_2 gauge transformation.

As a matter of fact, the decomposition formula (14) can further be simplified into a more elucidated expression [9]. The simplification procedure is shown below. If we introduce the Maurer–Cartan one-form

$$\begin{aligned}
R &= \frac{1}{\mathrm{i}}\,g^{-1}\,\mathrm{d}g\\
&= R^i\,T_i + R^a\,T_a + \tilde{R}^a\,t_a,
\end{aligned} \tag{15}$$

where $g \in G_2$, then the equation (14) can be rewritten using (13) as

$$A = g\,\underline{A}\,g^{-1} + \frac{1}{\mathrm{i}}\,\mathrm{d}g\,g^{-1}. \tag{16}$$

Here, the field $\underline{A}$ is another G_2 connection one-form that is gauge equivalent to the original one-form A (14). Explicitly, it takes the form

$$\underline{A} = \left(C^i - R^i\right)T_i - \frac{2}{3}\,\tilde{R}^a\,t_a + (\text{C.P.}), \tag{17}$$

where $(\text{C.P.}) = g^{-1}(\text{covariant part})g$.

Note that the (C.P.) space coincides with the orbit $G_2/U(1)^2$, and its local basis is spanned by 12 independent gauge covariant Lie-algebra valued one-forms. In addition, these covariant one-forms need to be orthogonal to the Cartan subalgebra T_i. To be more specific, we notice that the commutator $[R, T_i]$ is a gauge covariant one-form and is also orthogonal to the Cartan generator T_j, because $\mathrm{Tr}\,([R, T_i]\, T_j) = 0$. Thus, the commutator $[R, T_i]$ can be regarded as a part of the basis states of the (C.P.) space. Once a covariant one-form that determines a part of the basis states is found, the entire basis states can be determined by applying the adjoint action

$$\delta^i\, v = [\, v, T_i\,] \tag{18}$$

to that covariant one-form successively. v is an arbitrary Lie-algebra valued element.

What are the 12 covariant Lie-algebra valued one-forms that span the root $G_2/U(1)^2$? It is found that these covariant one-forms have much nicer expressions if we purposely separate the commutator,

$$[\, R, T_i\,] = R^a\, (\mathcal{F}_i)_{ab}\, T_b - \frac{1}{\sqrt{3}}\, \tilde{R}^a\, \epsilon_{ij}\, (\mathcal{F}_j)_{ab}\, t_b\,, \tag{19}$$

into two independent one-forms, denoted by X_i and x_i,

$$X_i \equiv R^a\, (\mathcal{F}_i)_{ab}\, T_b\,, \tag{20}$$

$$x_i \equiv -\frac{1}{\sqrt{3}}\, \tilde{R}^a\, \epsilon_{ij}\, (\mathcal{F}_j)_{ab}\, t_b\,. \tag{21}$$

Then, as mentioned in the above paragraphs, the use of the adjoint action (18) respectively on the X_i and x_i should render the entire basis states for the (C.P.) space. After some computations, we find that [9]

$$\delta^j X_i = Z_{ij}\,, \tag{22}$$

$$\begin{aligned}\delta^k Z_{ij} = {}& \frac{1}{3}\, (\,\delta_{ik}\, X_j + \delta_{jk}\, X_i) \\ & - \frac{1}{4}\, (\, d_{ijl}\, d_{lkm} - d_{ikl}\, d_{ljm} - d_{jkl}\, d_{lim})\, X_m \\ & - \frac{1}{4}\, (\, d_{ijl}\, Y_{lk} + d_{ikl}\, Y_{lj} + d_{jkl}\, Y_{li})\,,\end{aligned} \tag{23}$$

$$\delta^k Y_{ij} = d_{jkl}\, Z_{il} - d_{ikl}\, Z_{jl}, \tag{24}$$

and

$$\delta^j x_i = z_{ij}\,, \tag{25}$$

$$\begin{aligned}\delta^k z_{ij} = {}& \frac{1}{9}\, (\,\delta_{ik}\, x_j + \delta_{jk}\, x_i) \\ & - \frac{1}{12}\, (\, d_{ijl}\, d_{lkm} - d_{ikl}\, d_{ljm} - d_{jkl}\, d_{lim})\, x_m \\ & - \frac{1}{12}\, (\, d_{ijl}\, \epsilon_{lm}\, y_{mk} + d_{ikl}\, \epsilon_{lm}\, y_{mj} + d_{jkl}\, \epsilon_{lm}\, y_{mi})\,,\end{aligned} \tag{26}$$

$$\delta^k y_{ij} = d_{jkl}\, \epsilon_{lm}\, z_{im} - d_{ikl}\, \epsilon_{lm}\, z_{jm}\,, \tag{27}$$

where (3) and (4) have been used. Besides X_i and x_i, there are four extra one-forms Z_{ij}, Y_{ij}, z_{ij}, and y_{ij} in the above equations. Their explicit expressions are

$$Z_{ij} = R^a\, (\mathcal{F}_i\, \mathcal{F}_j)_{ab}\, T_b, \tag{28}$$

$$Y_{ij} = R^a\, (\mathcal{F}_i\, \mathcal{D}_j - \mathcal{F}_j\, \mathcal{D}_i)_{ab}\, T_b\,, \tag{29}$$

$$z_{ij} = \frac{1}{3}\, \tilde{R}^a (\tilde{\mathcal{F}}_k\, \tilde{\mathcal{F}}_l)_{ab}\, t_b\,, \tag{30}$$

$$y_{ij} = -\frac{1}{\sqrt{3}}\, \tilde{R}^a (\tilde{\mathcal{F}}_k\, \tilde{\mathcal{D}}_l - \tilde{\mathcal{F}}_l\, \tilde{\mathcal{D}}_k)_{ab}\, t_b\,, \tag{31}$$

where we define $(\tilde{\mathcal{F}}_i)_{ab} = \epsilon_{ij} (\mathcal{F}_j)_{ab}$ and $(\tilde{\mathcal{D}}_i)_{ab} = \epsilon_{ij} (\mathcal{D}_j)_{ab}$.

The outcome is that we get two independent subsets of Lie-algebra valued one-forms (X_i, Z_{ij}, Y_{ij}) and (x_i, z_{ij}, y_{ij}). Each of these separately forms a closed subalgebra under the adjoint action (18). These one-forms possess definite properties under the $SO(2)$ symmetry. For example, the one-forms X_i and x_i yield the $SO(2)$ vector representation, Z_{ij} and z_{ij} the $SO(2)$ symmetric tensor representation, and Y_{ij} and y_{ij} the $SO(2)$ antisymmetric tensor representation, i.e., the scalar representation. Accordingly, the number of independent components carried by the first set of one-forms (X_i, Z_{ij}, Y_{ij}) is 6, whereas the number carried by the second set (x_i, z_{ij}, y_{ij}) is also 6. The sum of independent one-forms in these two sets is 12, which matches the dimension of the space $G_2/U(1)^2$. Thus, the one-forms (x_i, z_{ij}, y_{ij}) and (X_i, Z_{ij}, Y_{ij}) can be used to parameterize the basis states of the (C.P.) space. Consequently, T_i, (x_i, z_{ij}, y_{ij}) and (X_i, Z_{ij}, Y_{ij}) all together yield a complete set of basis states for the G_2 Lie algebra.

To proceed to the complete decomposition of the G_2 connection (17), we need appropriate dual variables that appear as coefficients to the one-forms (x_i, z_{ij}, y_{ij}) and (X_i, Z_{ij}, Y_{ij}). We observe that the Yang–Mills connection $\underline{A}$ in (17) is a G_2 Lie-algebra valued one-form and transforms in the scalar representation of the $SO(2)$ symmetry. So, to form invariant combinations, the dual variables must be the Lie-algebra valued zero-forms and transform in the same $SO(2)$ representations as the associated covariant one-forms. Let us denote the dual variables by (ϕ^{ij}, ψ^{ij}) and (Φ^{ij}, Ψ^{ij}). Then the G_2 gauge field $\underline{A}_\mu$ (17) admits this Abelian decomposed expression:

$$\begin{aligned}\underline{A}_\mu \equiv {}& \underline{A}^i_\mu\, T_i + \underline{\tilde{A}}^a_\mu\, t_a + \underline{A}^a_\mu\, T_a \\ = {}& \left(C^i_\mu - R^i_\mu\right) T_i \\ & + \tilde{R}^a_\mu \left(\left(\phi^{ij} - \frac{2}{3}\delta_{ij}\right) (\tilde{\mathcal{F}}_i\, \tilde{\mathcal{F}}_j)_{ab} + \psi^{ij} (\tilde{\mathcal{F}}_i\, \tilde{\mathcal{D}}_j)_{ab} \right) t_b \\ & + R^a_\mu \left(\Phi^{ij} (\mathcal{F}_i\, \mathcal{F}_j)_{ab} + \Psi^{ij} (\mathcal{F}_i\, \mathcal{D}_j)_{ab}\right) T_b\,,\end{aligned} \tag{32}$$

with $(\tilde{\mathcal{F}}_i\, \tilde{\mathcal{F}}_i)_{ab} = \delta_{ab}$. Here Φ^{ij} and ϕ^{ij} are dual to the one-forms Z_{ij} and z_{ij}, respectively. Ψ^{ij} is dual to the one-forms X_i and Y_{ij}, and similarly ψ^{ij} is dual to the x_i and y_{ij}. Both duals, Ψ^{ij} and ψ^{ij}, decompose respectively into a vector and an antisymmetric tensor under $SO(2)$ symmetry. It is noted that the Abelian decomposed gauge field (32) contains the correct number of independent variables, and can be used for describing the G_2 gauge theory in the infrared limit.

Using (32), the G_2 field strength tensor under the Abelian projection reads

$$\begin{aligned}F_{\mu\nu} = {}& \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu - \mathrm{i}\, [\underline{A}_\mu, \underline{A}_\nu] \\ = {}& \Bigg[\left(C^i_{\mu\nu} - R^i_{\mu\nu} + f_{iab}\, \underline{A}^a_\mu\, \underline{A}^b_\nu\right) \delta_{ij} \\ & + \frac{1}{\sqrt{3}}\, f_{iab}\, \epsilon_{ij}\, \underline{\tilde{A}}^a_\mu\, \underline{\tilde{A}}^b_\nu \Bigg] T_j\end{aligned}$$

$$
\begin{aligned}
&+\Bigg[(\tilde{D}_\mu)_{ac}\,\tilde{\underline{A}}_\nu^a-(\tilde{D}_\nu)_{ac}\,\tilde{\underline{A}}_\mu^a+\frac{2}{\sqrt{3}}\,f_{abc}\,\tilde{\underline{A}}_\mu^a\,\tilde{\underline{A}}_\nu^b\\
&\quad -g_{acb}\left(\tilde{\underline{A}}_\mu^a\,\underline{A}_\nu^b-\tilde{\underline{A}}_\nu^a\,\underline{A}_\mu^b\right)\Bigg]t_c\\
&+\Bigg[(D_\mu)_{ac}\,\underline{A}_\nu^a-(D_\nu)_{ac}\,\underline{A}_\mu^a+f_{abc}\,\underline{A}_\mu^a\,\underline{A}_\nu^b\\
&\quad +g_{abc}\,\tilde{\underline{A}}_\mu^a\,\tilde{\underline{A}}_\nu^b\Bigg]T_c\,, \qquad (33)
\end{aligned}
$$

where $\underline{A}_\mu^a$ and $\tilde{\underline{A}}_\mu^a$ are given in (32). In addition, we denote $C_{\mu\nu}^i=\partial_\mu C_\nu^i-\partial_\nu C_\mu^i$ and $R_{\mu\nu}^i=\partial_\mu R_\nu^i-\partial_\nu R_\mu^i$. The $U(1)$ covariant derivatives in (33) are defined as

$$
\begin{aligned}
(D_\mu)_{ab}&=\partial_\mu\,\delta_{ab}+f_{iab}\left(C_\mu^i-R_\mu^i\right)\,,\\
(\tilde{D}_\mu)_{ab}&=\partial_\mu\,\delta_{ab}-\frac{1}{\sqrt{3}}\,f_{iab}\,\epsilon_{ij}\left(C_\mu^j-R_\mu^j\right)\,. \qquad (34)
\end{aligned}
$$

Therefore, the low-energy effective lagrangian of the G_2 gauge theory is ready to be written down if we substitute the decomposed field strength tensor (33) into the Yang–Mills theory, $\mathcal{L}_{G_2}=(1/2g^2)\,\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$. Here, g is the Yang–Mills coupling constant. The resultant low-energy lagrangian will become a non-renormalizable theory involving various fields, such as C_μ^i, R_μ^a, Ψ^{ij}, ϕ^{ij}, etc. However, we shall not present this lagrangian in any detail. Instead, we are interested in a particular low-energy phase of this non-renormalizable lagrangian. To this aim, let us consider the dynamical fields R_μ^i, R_μ^a, and $\tilde{R}_\mu^a$ living in the classical field backgrounds $\langle C_\mu^i\rangle=0$, $\langle\Phi^{ij}\rangle=\langle\Psi^{ij}\rangle=\langle\psi^{ij}\rangle=0$, and $\langle\phi^{ij}\rangle=(2/3)\,\delta_{ij}$. Then these classical fields can be properly integrated out with the conditions $\langle\partial_\mu\Phi^{ij}\partial_\nu\Phi^{lk}\rangle\sim\langle\partial_\mu\Psi^{ij}\partial_\nu\Psi^{lk}\rangle\sim\langle\partial_\mu\phi^{ij}\partial_\nu\phi^{lk}\rangle\sim\langle\partial_\mu\psi^{ij}\partial_\nu\psi^{lk}\rangle\sim g_{\mu\nu}$. After performing the field integration, the theory takes the form

$$
\mathcal{L}_{G_2}=M^2\,R_\mu^a\,R_\mu^a+m^2\,\tilde{R}_\mu^a\tilde{R}_\mu^a+\frac{1}{4g^2}\left(\partial_\mu R_\nu^i-\partial_\nu R_\mu^i\right)^2\,, \qquad (35)
$$

where M and m are some constants with the dimension of a mass. The model (35) indeed represents the G_2 generalization of the original Faddeev–Skyrme model and is believed to be relevant to the infrared limit of the G_2 Yang–Mills theory. It would be interesting to understand the detailed structure of the model.

In conclusion, we have systematically studied the G_2 algebra based on the structure of its regular maximal subalgebra, an $SU(3)$ algebra. The structure constants of the G_2 algebra are established neatly using the same $SU(3)$ symmetry. We have also derived the low-energy lagrangian of the G_2 gauge theory using the Abelian decomposition method proposed by Faddeev and Niemi. The low-energy theory is shown to be the G_2 generalization of the Faddeev–Skyrme model.

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References

1. Y. Nambu, Phys. Rev. D **10**, 4262 (1974); S. Mandelstam, Phys. Rep. C **23**, 245 (1976); A. Polyakov, Nucl. Phys. B **120**, 429 (1977)
2. G. 't Hooft, Nucl. Phys. **153**, 141 (1979); B **190**, [FS], 455 (1981)
3. L.D. Faddeev, A.J. Niemi, Phys. Rev. Lett. **82**, 1624 (1999), hep-th/9807069
4. L.D. Faddeev, A.J. Niemi, Phys. Lett. B **525**, 195 (2002), hep-th/0107187
5. L.D. Faddeev, A.J. Niemi, Nature (London) **387**, 58 (1997), hep-th/9610193; R. Battye, P. Sutcliffe, Phys. Rev. Lett. **81**, 4798 (1998), hep-th/9808129
6. K. Konishi, K. Takenaga, Phys. Lett. B **508**, 392 (2001), hep-th/9911097; T. Tsurumaru, I. Tsutsui, Nucl. Phys. B **589**, 659 (2001), hep-th/0005064
7. W.-C. Su, Phys. Lett. B **525**, 201 (2002), hep-th/0108047; Chin. J. Phys. **40**, 516 (2002), hep-th/0107187
8. V. Periwal, hep-th/9808127
9. L. Faddeev, A.J. Niemi, Phys. Lett. B **449**, 214 (1999), hep-th/9812090; B **464**, 90 (1999), hep-th/9907180
10. W.-C. Su, Phys. Lett. **499**, 275 (2001), hep-th/0009161; Eur. Phys. J. C **20**, 717 (2001), hep-th/0008093
11. Y.M. Cho, Phys. Rev. D **21**, 1080 (1980); D **23**, 2415 (1981); Phys. Rev. Lett. **44**, 1115 (1980)